H Algebra 3
Name:
More Expected Value in ...

## You need to show work to justify your answers-no naked numbers!

1. Car Insurance. To calculate the premium each insured must pay, the pricing actuary needs to calculate the expected number of claims each driver will have over the 6 month period and the expected amount of those claims. The pricing formula then is:

$$
\text { Premium }=E(\# \text { of claims }) * E(\text { amount of claim })
$$

Now let's assume that the pricing actuary has the following distribution for the number of claims for a driver in the 6-month period covered by the policy, also known as the frequency:

| Number of Claims | Probability |
| :---: | :---: |
| 0 | $50 \%$ |
| 1 | $25 \%$ |
| 2 | $15 \%$ |
| 3 | $10 \%$ |

a. What is the probability that someone has more than 3 claims in the 6-month period? Justify your response.
b. With this distribution, what is the expected value of the number of claims?

Next, assume the pricing actuary has the following distribution for the severity, or cost for each claim.

| Cost of Claim | Probability |
| :---: | :---: |
| $\$ 5,000$ | $10 \%$ |
| $\$ 10,000$ | $40 \%$ |
| $\$ 15,000$ | $45 \%$ |
| $\$ 20,000$ | $5 \%$ |

c. With this distribution, what is the expected value of the cost of each claim?
d. Using the formula, above, calculate the premium the company should charge.

In the next year, the pricing actuary does another study and finds the probabilities of the number of accidents and the amount per accident as follows.

| Number of claims | Probability |
| :--- | :--- |
| 0 | $40 \%$ |
| 1 | $35 \%$ |
| 2 | $10 \%$ |
| 3 | $10 \%$ |
| 4 | $5 \%$ |


| Cost of Claim | Probability |
| :--- | :--- |
| $\$ 5000$ | $20 \%$ |
| $\$ 10000$ | $50 \%$ |
| $\$ 15000$ | $20 \%$ |
| $\$ 20000$ | $10 \%$ |

e. Recalculate the premium the company should charge.
2. Life Insurance. For a particular age group, statistics show that the probability of dying in any one year is 1 in 1000 people and the probability of suffering some sort of disability is 3 in 1000 people. The West Life Insurance Company offers to pay out $\$ 20000$ if you die and $\$ 10000$ if you are disabled. What profit is the insurance company making per customer based on the expected value if it charges a premium of $\$ 100$ to its customers for the above policy.
3. Farming. High school students Jennifer and Lucas Harris would like to devote some acreage from their thriving new vegetable business to heirloom tomatoes, which they've noticed are big sellers at the farmers' market. "There are some juicy varieties that look great and taste even better!" exclaimed Lucas. "But with all those varieties, how will we decide which seeds to invest in?" Jennifer wondered. Jennifer and Lucas met with Ray "Red" Vines, the local representative of the Herbivore Seed Company. "Well," said Red, "I can't predict which tomato variety will give the biggest yield in the future, but I can show you data on how my customers have done over the past four years with my two best-selling seeds."

Tomato Seed Yield (Tons per Acre)

| Tomato Variety | Year | Farm 1 | Farm 2 | Farm 3 | Farm 4 | Farm 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Red Ready | 2009 | 19 | 15 | 11 | 14 | 20 |
|  | 2010 | 16 | 18 | 16 | 16 | 16 |
|  | 2011 | 10 | 19 | 11 | 12 | 16 |
|  | 2012 | 14 | 12 | 20 | 16 | 18 |
| Crimson Champ | 2009 | 26 | 0 | 12 | 20 | 17 |
|  | 2010 | 0 | 14 | 10 | 26 | 28 |
|  | 2011 | 30 | 28 | 16 | 17 | 26 |
|  | 2012 | 10 | 16 | 17 | 10 | 17 |

a. Prepare a probability distribution table for each variety of tomato.

Red Ready

| Number of Tons | Probability |
| :---: | :---: |
| 10 |  |
| 11 |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Crimson Champ

| Number of Tons | Probability |
| :---: | :---: |
| 0 |  |
| 10 |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

b. Calculate the expected value for each variety.
c. Which seeds would you recommend Jennifer and Lucas invest in?
4. Games of Chance. Two dice are rolled. If the total showing is a prime number, you pay your friend $\$ 6$; otherwise, your friend pays you $\$ 2$.

|  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | $\mathbf{7}$ | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

a. What is the expected value of the game?
b. If you played the game 40 times, what are your expected winnings?
c. How much should you pay your friend when you lose so that your expected winnings are exactly \$0?
5. Casino Game. In a casino game, the average payout (expected value) for a player is $\$ 2.53$. A partially completed probability model for this game is given below.

TABLE 1.5: Casino Game Payouts

| Amount Paid | $\$ 0$ | $\$ 1$ | $\$ 3$ | $? ? ?$ | $\$ 21$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.32 | 0.47 | 0.08 | 0.07 | 0.06 |

a. What is the missing amount?
b. If the casino were going to set a price for this game, do you think they would choose to charge $\$ 2$ or $\$ 3$ ? Explain your choice.
c. If the casino were going to set a price for this game, do you think they would choose to charge $\$ 3$ or $\$ 6$ ? Explain your choice.
6. Vehicle Ownership. The table below represents the number of vehicles and the associated probability of having that number of vehicles in an individual household. What is the expected number of vehicles in a typical household?

TABLE 1.3: Vehicle Ownership

| \# Owned | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probability | 0.02 | 0.26 | 0.37 | 0.19 | 0.12 | 0.04 |

7. Fundraising. A student sells products as part of a fundraiser to raise money for a choir trip to New York. She sold 75 items total, including 50 rolls of cookie dough for $\$ 6$ each, 15 packages of butter braids at $\$ 10$ each, and 10 bake-at-home bread packs for $\$ 12$ each.
a. Build a probability distribution table for this situation.
b. What is the expected value of a sale for this particular student?
c. How much money did she raise?
